FRACTAL MODELING OF CLOUD-TO-GROUND LIGHTNING

Whitney Tidwell*, Jérémy A. Riousset†, and Victor P. Pasko‡

CSSL Laboratory, Department of Electrical Engineering
The Pennsylvania State University, University Park, PA 16802

*Undergraduate student of
Department of Physics
Rhodes College, Memphis, TN 38112

ABSTRACT

Marshall and Stolzenburg[1] hypothesized following discussion by Kasemir[2] that lightning, as a mover of charge within the thundercloud[3, 4], is responsible for decreases in the total electrostatic energy of a thunderstorm. Therefore, understanding the mechanisms of lightning propagation can lead to a better understanding of the redistribution of charge within the thundercloud. Niemeyer *et al.*[5] suggested that gas discharges can be modeled using a probabilistic approach based on Mandelbrot's[6] fractal theory. Following this description, several fractal models of lightning discharge have been developed[7–9]. The results presented in this paper are obtained using the three-dimensional fractal model of lightning discharge discussed by Riousset *et al.*[10] and Riousset[11]. This model combines the Niemeyer et al. hypothesis with the assumption put forth by Kasemir[2] that lightning discharges are equipotential and overall neutral. We apply this model to investigate cloud-to-ground discharges and related cloud charge configurations leading to this types of discharge. Results are compared with measurements of actual events similar to those reported in Coleman *et al.*[12] obtained using the Lightning Mapping Array (LMA) observing lightning discharges over Langmuir Laboratory, New Mexico.

†Graduate Mentor
‡Faculty Mentor

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

INTRODUCTION

The search for a theoretical model to describe lightning propagation began in the 1950s[2]. In particular, Kasemir[2] hypothesized that the lightning channel is both equipotential and overall neutral. This idea follows from the description of the lightning channel as a plasma wave of mainly leader nature, which had been established as early as the 1930s by Schonland, Malan, and their co-workers during their photographic studies of the initiation and propagation of cloud-to-ground lightning in South Africa[13]. However, an understanding of the leader process is still far from complete[14–16]. Leaders, due to their high conductivity, are analogous to an equipotential metallic wire that becomes polarized when placed in a non-zero electric field. Hence, charge accumulates at the tip of the leader and enhances the surrounding electric field above the threshold required for the initiation of streamers. As a result, a streamer zone develops around the leader tip. In the streamer zone, streamers are generated at a frequency of about 10^9 s^{-1}. Consequently, the sum of these numerous streamer channel currents leads to heating of the region ahead of the leader tip, leading in turn to an increase of the conductivity of the heated region, which eventually allows further leader propagation[10]. It should be noted that processes occurring in the streamer zone are not fully understood at present, therefore, most existing models do not attempt to model them directly.

Interest in lightning modeling was recently renewed by the potential hazards posed by lightning strokes to aircraft, spacecraft, and installations using solid-state electronics. Also, major advances in lightning modeling were made possible by the development of computer science in the 1970s and 1980s. Some early computer models of lightning focused on the interaction between the lightning channel and the surrounding thunderstorm electric field[17, 18], while others focused on only bulk effects of the discharge[19]. In most of these early models, the lightning channel propagates along electric field lines and does not exhibit the highly branched behavior of the lightning channel that is often observed in nature. Figure 1 shows a cloud-to-ground flash that took place over Arecibo, Puerto Rico. It should be noted that the flash initiation occurs in the lower part of the cloud, where the channel appears brightest. The channel develops branches that propagate both downward toward the ground and upward into the cloud (not visible in this picture but measured by the Lightning Mapping Array (LMA), see Figure 2 and further discussion in this paper for details).

At present, a lightning model based on a microphysical approach to the mechanisms governing lightning initiation and propagation is not possible due to both the lack of a complete theory of the involved processes and of computational power to model these processes. Therefore, Niemeyer *et al.*[20] attempted to model gas discharges on a macroscopic scale. To overcome the limitations of the earlier deterministic models, they developed a probabilistic model based on Mandelbrot's[6]

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

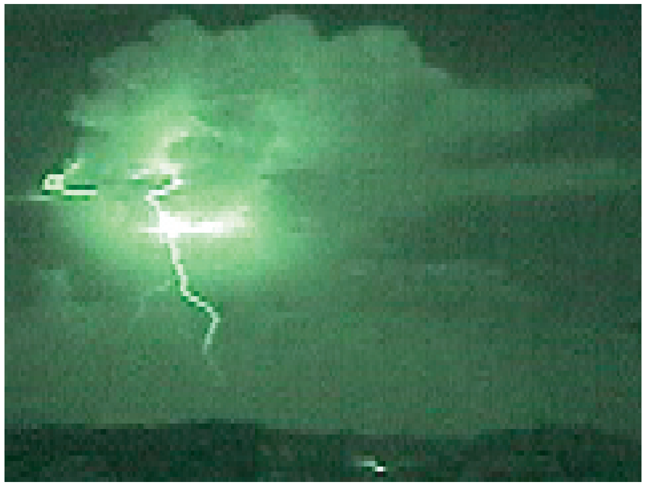

Figure 1: A cloud-to-ground flash over Arecibo, Puerto Rico. The flash initiates in the lower part of the cloud where the channel appears brightest and develops channels that propagate in the downward and upward directions.

fractal theory. The model discharge propagates in two dimensions, and the probability for a path to be chosen is calculated based on both the ambient electric potential and the electric potential along the existing discharge channel. A path is then randomly chosen among all the candidate links for propagation. Later, more lightning-specific models such as Petrov and Petrova[21] were developed, which revisited the fractal model and applied it to intracloud discharges, cloud-to-ground discharges and ground-to-cloud discharges. Other fractal models of lightning were introduced in the context of the cloud electrification model[8], or were applied to investigate the probability of a strike to a structure[9]. However, none of these models ensured overall neutrality of the discharge by adequately shifting the channel potential. Therefore, Riousset *et al.*[10] recently applied a three-dimensional fractal model to simulate intracloud discharges. This model combines Niemeyer *et al.*'s[20] fractal approach and Kasemir's[2] hypotheses to create a model of lightning that is stochastic, equipotential, and overall neutral.

The goal of this paper is to apply Riousset *et al.*'s[10] three-dimensional fractal model in order to investigate cloud-to-ground discharges and cloud charge configurations leading to this type of discharge. Cloud charge configurations are based on the common tripole hypothesis[4, 22], which is discussed and detailed further in this paper. Results will be compared with measurements of an actual event by Coleman *et al.*[12] obtained using the Lightning Mapping Array (LMA) operating over Langmuir Laboratory, New Mexico (Figure 2) and with other relevant data available in the referred literature.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

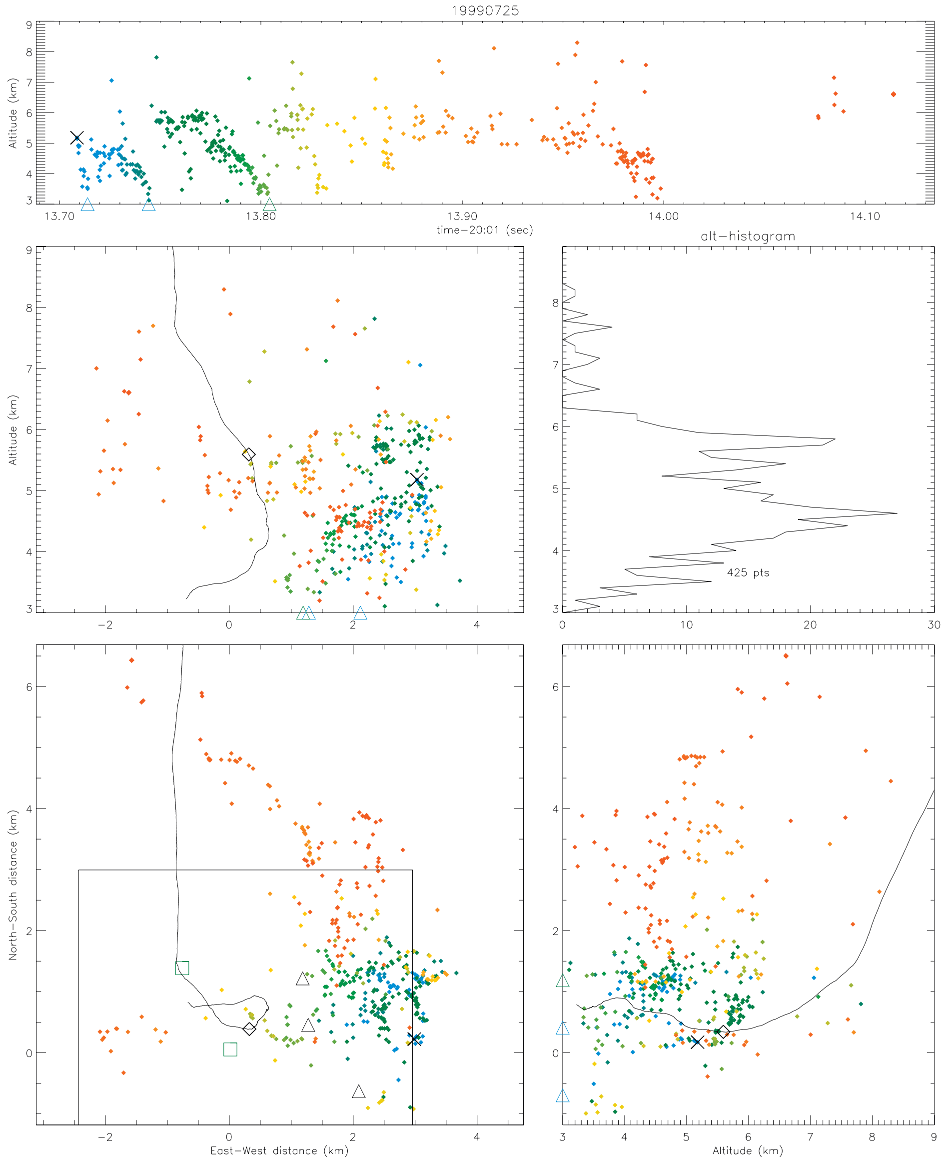

Figure 2: Flash F from Coleman *et al.*[12]. This cloud-to-ground flash occurred at 2001:13 UT on July 25 1999. From the altitude of initiation, it can be guessed that this flash is of negative polarity. In the original figure, the occurrence of radiation sources which trace the lightning path[23] are color-coded with the first sources colored blue and the last sources colored red. Shown are five different graphs (clockwise from the top): altitude versus time, altitude histogram of the sources in 100-m bins, a projection onto the north-south vertical plane, a horizontal projection of the sources, and a projection onto the west-east vertical plane. The cross denotes the position of the first Lightning Mapping Array (LMA) source. Triangles indicate three ground strike locations detected by the National Lightning Detection Network. The path of an instrumented balloon is shown in the projections; the diamonds show the location of the balloon at the time of the flash.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

MODEL FORMULATION

The lightning channel and thundercloud are modeled in a three-dimensional Cartesian coordinate system whose exact dimensions are discussed at the end of this section. The cloud electrical structure used for this model is based on the classical tripole structure[4, 22], the validity of which has since been confirmed by several authors[12, 19, 24]. The core of the tripole model consists of two main charge layers: a dominant central negative charge layer below (N), and an upper positive layer of charge (P) with similar and opposite charge content[22]. Below the main charge layers exists a layer of positive charge (LP) of lesser magnitude than the upper layers, completing the tripole. A negative screening layer is sometimes added above the upper positive charge layer[19], but is not implemented in this study. Assuming that the ground is a perfect electrical conductor, three additional image charges must be accounted for. The ambient electric field of the thunderstorm can be solved numerically using the successive overrelaxation (SOR) method[10].

It has been hypothesized that different cloud charge configurations lead to cloud-to-ground lightning of different polarity[3]. Negative cloud-to-ground lightning can be produced from a tripole structure whose charge layers are centered on the same vertical axis[4]. The charge configuration used to reproduce negative cloud-to-ground lightning for this model represents a non-uniform charge configuration which was approximated by a Gaussian distribution with parameters summarized in Table I. The values for the upper positive layer, the central negative layer, and the lower positive layer were respectively 50 C, −60 C, and 13 C (also shown in Table I). These values were based on experimental data found and presented by Krehbiel *et al.*[19]. Figure 3 shows the charge density and the electric field lines produced by this cloud charge configuration. The tripole structure can clearly be seen in this plot: light shading indicates regions of positive charge while dark shading represents regions of negative charge. The black arrows represent electric field lines.

Less is known about the production of positive cloud-to-ground lightning. Jursa's[3] diagram of the typical charge distribution and lightning patterns of a mid-latitude thunderstorm pictures a tripole structure in which the upper positive charge layer is extended significantly out over the ground, forming a tilted electrical structure. Such a structure is thought to allow positive cloud-to-ground strikes to bypass the negative and lower positive charge layers and go straight to the ground. Positive polarity lightning may also be produced by a charge configuration in which all three charge layers are aligned, but the negative charge layer has been depleted, as during the final stages of a thunderstorm's life[4, 25, 26]. The model of positive cloud-to-ground lightning presented in this paper combines these two hypotheses. The cloud charge layers are represented by a non-uniform charge configuration which was approximated using Gaussian distributions with parameters given in Table I.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

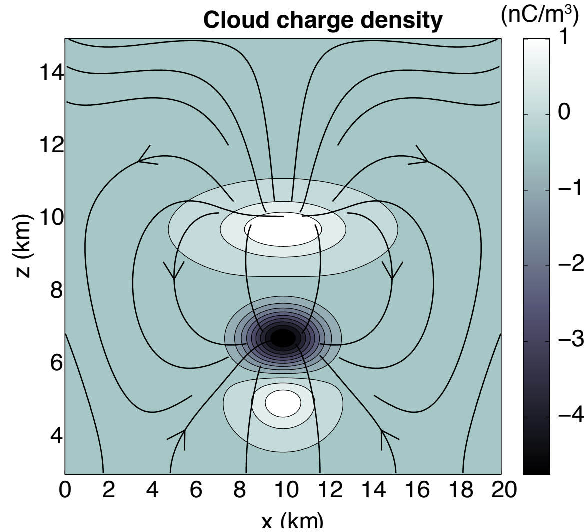

Figure 3: Cloud charge density and electric field lines produced by a cloud charge configuration leading to negative cloud-to-ground lightning. Light shading indicates regions of positive charge while dark shading represents regions of negative charge. The black arrows represent electric field lines.

All three layers were centered in the y-z plane, but the center of the upper positive charge layer was displaced by 8 km in the x-direction. The net charges contained in the upper positive, central negative, and lower positive layers were respectively 60 C, -40 C, and 8 C (also shown in Table I). Figure 4 shows the charge density and electric field lines produced by this cloud charge configuration.

The field and potential distributions induced by cloud charges are derived assuming open boundary conditions at the side and top boundaries of the simulation domain. The bottom boundary represents the ground and is assumed to be a perfect conductor with zero potential. The potential at the boundaries is found using the following equation[27]:

$$\phi(\vec{r}) = \phi_{amb}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\iiint_{V'} \frac{\rho_{amb}(\vec{r'})}{|\vec{r}-\vec{r'}|}dV' + \frac{1}{4\pi\epsilon_0}\iiint_{V'} \frac{\rho^{i}_{amb}(\vec{r'_i})}{|\vec{r}-\vec{r'_i}|}dV' \quad (1)$$

where $\vec{r}$ represents the coordinate vector of a point on the boundary and $\phi_{amb}(\vec{r})$ the potential at this point. $\rho_{amb}(\vec{r'})$ represents the ambient charge density at a point $\vec{r'}$, and $\rho^{i}_{amb}(\vec{r'})$ represents the charge density of the image charges at point $\vec{r'}$. After the potential at the boundaries is found, the ambient potential is derived by solving Poisson's equation $\nabla^2\phi_{amb} = -\rho_{amb}/\varepsilon_0$ using the SOR algorithm, and the electric field is obtained by finite differentiation[10].

The exact value for lightning initiation and propagation threshold has yet to be accurately determined, therefore we assume the initiation and propagation thresh-

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

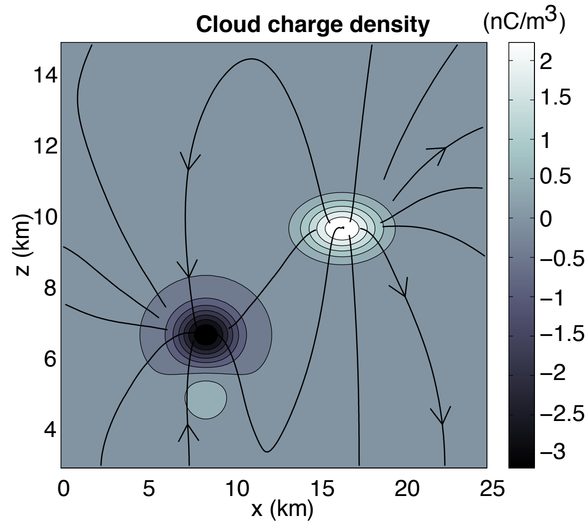

Figure 4: Cloud charge density and electric field lines produced by a cloud charge configuration leading to positive cloud-to-ground lightning. Light shading indicates regions of positive charge while dark shading represents regions of negative charge. The black arrows represent electric field lines.

Table I: Parameters of the charge layers. Adapted from Krehbiel *et al.*[19].

Layer	$x_Q\,(\mathrm{km})^a$	$y_Q\,(\mathrm{km})^a$	$z_Q\,(\mathrm{km})^{a,d}$	$a_x\,(\mathrm{km})^b$	$a_y\,(\mathrm{km})^b$	$a_z\,(\mathrm{km})^a$	$Q\,(\mathrm{C})^c$
			For negative cloud-to-ground discharge				
LP	10.0	10.0	5.00	1.50	1.50	0.75	13.0
N	10.0	10.0	6.75	1.73	1.73	0.75	-60.0
P	10.0	10.0	9.75	2.82	2.82	0.75	40.0
			For positive cloud-to-ground discharge				
LP	4.5	12.5	5.00	1.50	1.50	0.75	8.0
N	4.5	12.5	6.75	1.73	1.73	0.75	-40.0
P	20.5	12.5	9.75	2.30	2.30	0.75	60.0

[a] Coordinates of the charge center
[b] Space scales of the Gaussian distribution
[c] Charge values
[d] Measured above sea level

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

olds to be $E_{init} = E_{th}^{\pm} = \pm 2.16$ kV/cm at sea level[28]. In addition, we know that these threshold values change with altitude[14]. Thus, the model uses the following expression to derive the initiation and propagation thresholds:

$$E_{init}(z) = E_{th}^{\pm}(z) = \pm 2.16 \frac{N(z + z_{gnd})}{N_0} [\mathrm{kV/cm}] \quad (2)$$

where z is the altitude above the ground, z_{gnd} is the altitude of the ground plane, N is the value of the neutral charge density, and N_0 is the value of N at sea level[10, 11].

The initiation point is chosen from regions of high electric field (i.e., exceeding the initiation threshold) at altitudes corresponding to the altitude of initiation of positive or negative cloud-to-ground discharges. From the initiation point, the lightning channel begins to propagate. Only one link is added at each step and the potential ϕ_0 along the channel is recalculated after each step to ensure neutrality of the channel[10]. First, the new candidates for propagation are identified. Any point within one grid step of the channel that is not at any boundary and that possesses a potential difference between its ends that exceeds the threshold for propagation is a viable candidate for the next stage of development. The difference in potential for each candidate link is calculated as $E_i = (\phi^{start} - \phi^{end})/l$, where ϕ^{start} and ϕ^{end} are the potentials at either end of the candidate link and l is its length.

The probability for a candidate being chosen for propagation is then calculated by[29]:

$$p_i = \frac{|E_i - E_{th}^{\pm}|^{\eta}}{\sum_i |E_i - E_{th}^{\pm}|^{\eta}} \quad (3)$$

where η describes the sensitivity of the probability to the field strength[7]. Following Niemeyer and Wiesmann[30] and Mansell *et al.*[8], η is chosen equal to 1. The next link defining discharge propagation is randomly chosen among the candidate links, accounting for their respective probabilities. The process is illustrated in a two-dimensional plane in Figure 5. The extension to three dimensions is straightforward. The solid lines in Figure 5a represent the existing channel, and the dashed lines represent candidate links. In Figure 5b, the probabilities for each candidate link are represented on a unit length segment, and a random point is chosen between 0 and 1. The candidate link corresponding to the section of the segment from which the random point is chosen will become the new section of the channel.

For further propagation to take place, the electric potential must be recalculated for every point within the domain to ensure the overall neutrality of the discharge. This is done using a procedure based on the bisection method and described by Riousset *et al.*[10] and Riousset[11].

This step-wise process continues until either a boundary is reached or the ambient electric field no longer exceeds the threshold field anywhere in the simulation domain. (It is important to note that the simulation will stop when the channel first reaches the ground, which corresponds only to the part of Figure 2 up to the first

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

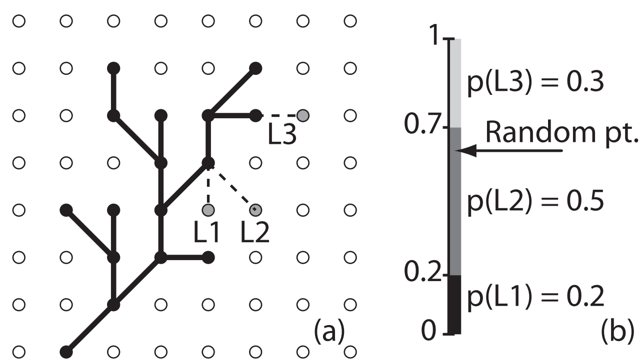

Figure 5: Channel extension in a 2-D geometry[10, 11]. (a) Channel links (solid lines) and link candidates (dashed lines); (b) Probability associated with each link. (The values of the probabilities given on this plot are arbitrary and are shown only for two representative points on the existing discharge tree for the purposes of illustration. Real values are derived based on the analysis of potential differences involving all grid points of the existing discharge tree–see text for details.)

return stroke around 20:01.13.71 UT. Therefore, subsequent strokes will not appear in the simulation results.)

For the negative cloud-to-ground discharge, the simulation domain is divided using rectangular grids with dimensions 0.5 km $\times$ 0.5 km $\times$ 0.15 km. Thus, the steps taken in the horizontal direction are of length 0.5 km while the steps taken in the vertical direction are of length 0.15 km. The size of the simulation domain in this case is 20 km $\times$ 20 km $\times$ 12 km. For the positive cloud-to-ground discharge, the simulation domain is discretized using 1.25 km long grid steps in the x- and y-directions and 0.3 km grid steps in the z-direction. The simulation domain extends to 25 km in the x- and y-directions and 12 km in the z-direction.

RESULTS

In this section, we present two simulation runs. First we discuss the model negative cloud-to-ground discharge, then we present the results for the model positive cloud-to-ground discharge.

Figure 6 shows a representative example of a model negative cloud-to-ground discharge leading up to the first stroke to ground. The steps are color-coded with the first steps colored dark grey and the last steps colored light grey. Panel (a) represents the altitude of each newly added link. Figure 6 also shows three views of the discharge tree: a projection in the x-z plane (panel (b)), x-y plane (panel (d)), and y-z plane (panel (e)). The figure shows a histogram of the number of grid points occupied by a link as a function of altitude (panel (c)). A triangle indicates the

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

point at which the stroke reaches the ground. The initiation point of this discharge is marked by a cross and is beneath the central negative layer at an altitude of 6 km above sea level (the ground has been set at an altitude of 3 km above sea level to simulate conditions over Langmuir Laboratory in New Mexico). Two sets of branches develops from the initiation point . The upper set of branches propagates horizontally within the central negative layer. The lower set of branches propagates rather horizontally between 4 km and 6 km through the lower positive charge layer before going straight to the ground.

Figure 7 shows that before the first stroke to the ground takes place the initiation and propagation thresholds are exceeded by $\sim$80% between the altitudes of 6.5 km and 9 km, which corresponds to the possible initiation of intracloud discharges, and at $\sim$6 km by $\sim$45%, which corresponds to the initiation of negative cloud-to-ground discharges. Strong field reduction is observed in the lower part of the plot in regions where the discharge propagated.

The total charge transferred by the channel was 23.3 C, and the total length of the channel was 17.2 km, thus the linear charge density of the channel was 1.35 mC/m.

Figure 8 shows a model positive cloud-to-ground discharge using the same formatting as in Figure 6. The discharge initiates at 9 km above sea level and propagates vertically to the ground with very little horizontal propagation tortuosity or branching.

The total charge transferred by the channel was 27.8 C and the total length of the channel was 8.75 km, thus the linear charge density of the channel was $\sim$3.2 mC/m.

DISCUSSION

It has been reported that in North America, the average ratio of cloud lightning to cloud-to-ground lightning is between 2.5 and 3[4], and of all cloud-to-ground lightning, about 90 percent is negative and about 10 percent is positive[4]. These statistics indicate that the occurrence of cloud-to-ground lightning of any polarity is not as frequent as the occurrence of intracloud discharges and the instance of a positive cloud-to-ground discharge is rare. The relative rarity of cloud-to-ground discharges compared to intracloud flashes becomes more meaningful when considering that both intracloud and cloud-to-ground discharges may originate from the same cloud charge structure[19]. Indeed, we found that the cloud configurations used in the model were usually capable of producing more than one type of discharge. For example, Figure 7 shows that this charge configuration can lead to both intracloud and negative cloud-to-ground discharges, consistent with Krehbiel *et al.*'s[19] study. Similarly, a closer look at the configuration shown in Figure 4 reveals that both negative cloud-to-ground and positive cloud-to-ground flashes are likely to occur.

Further investigation revealed that both positive and negative cloud-to-ground

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

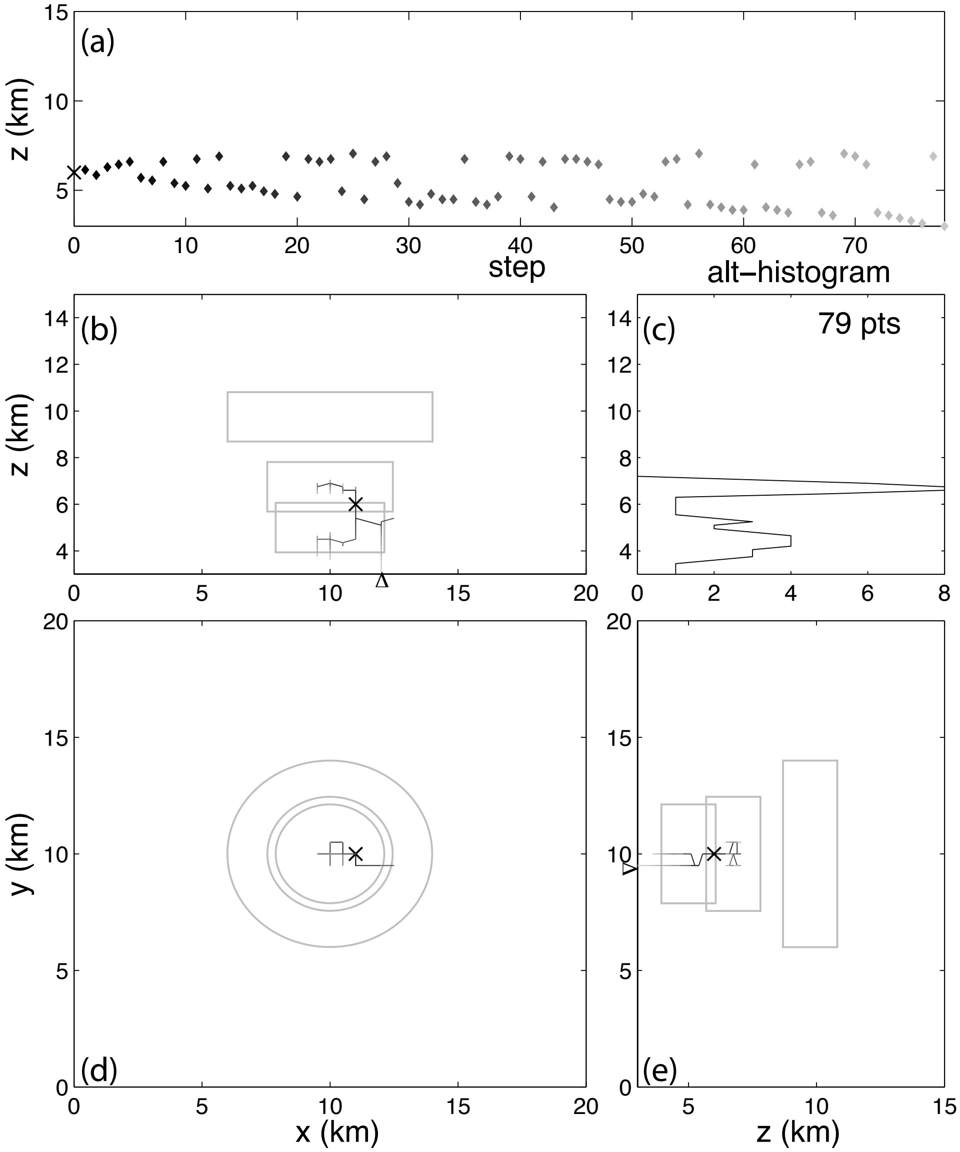

Figure 6: An example of a model negative cloud-to-ground discharge leading to the first stroke to ground. The steps are color-coded with the first steps colored dark grey and the last steps colored light grey. Panel (a) represents the altitude of each newly added link. Shown are three views of the discharge tree: a perspective from (b) the x-z plane, (d) the x-y plane, (e) the y-z plane. (c) shows a histogram of the grid point occupied by a channel link taken as a function of altitude. The upper positive, central negative, and lower positive charge layers are shown as cylinders with diameter $2a_x$ and depth $2a_z$ outlined in grey. The initiation point of this discharge is beneath the upper positive layer and is marked by a cross. A triangle indicates the point at which the lightning tree reaches the ground.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

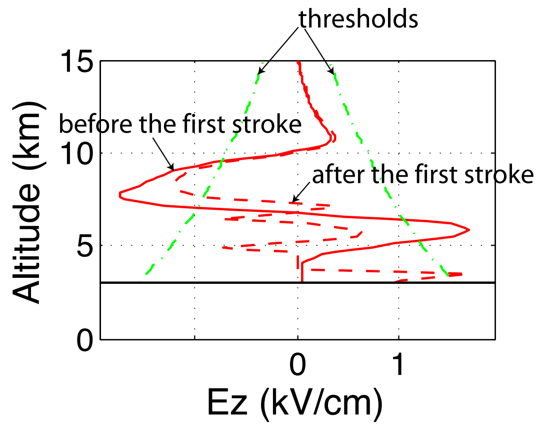

Figure 7: Electric field profiles measured on the central axis of the simulation domain before and after the occurrence of the negative model's first stroke to the ground. The dot-dashed lines represent the propagation thresholds. The solid line represents the total electric field before the first stroke to the ground and the dashed line represents the total electric field after the first stroke to the ground.

flashes were more readily produced by non-uniform charge layers than by disk-shaped layers of uniformly distributed charge. Thus, a Gaussian representation of the charge layers was chosen. In general, the uniform charge layers required about twice as much charge as the non-uniform layers to exceed the initiation and propagation thresholds. For the same net charge, the use of a Gaussian distribution creates regions of higher charge density and lower charge density. In the former, the electric field can be sufficiently high to initiate the discharge, but the field will fade rapidly in the region of lower density. Physically, it is expected that lightning discharges are initiated in such small regions of high electric field[11].

A quantification of the performance of the model can be achieved by comparing the channel parameters (in particular altitude of initiation, charge transfer, length, and linear charge density) with measurements available from the available literature.

A comparison of Figures 2 and 6 shows similar altitudes of initiation of $\sim$6 km (i.e., below the main negative charge center) for a negative cloud-to-ground stroke. Williams *et al.*[31] first showed that a leader channel of positive or negative polarity will propagate in a region of negative or positive charge, respectively. This behavior has also been observed for actual intracloud discharges[12] and modeled using numerical simulation[10]. Therefore, it is expected that lightning leaders propagate in regions of opposite polarity. Consistent with the above discussion and Figure 2, the portion of the negative cloud-to-ground discharge developing in the negative charge layer is positive and the lower part which reaches the ground is negative.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

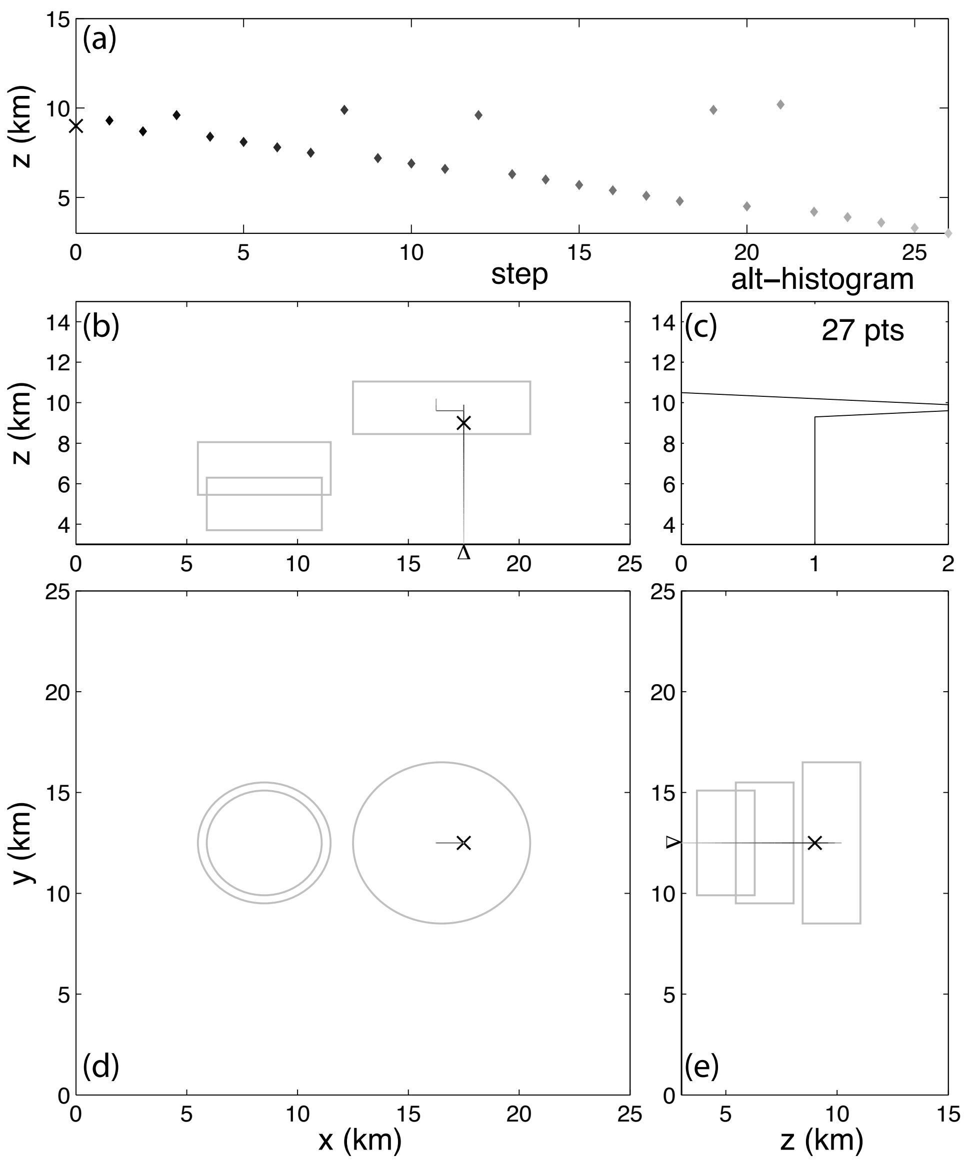

Figure 8: An example of a model positive cloud-to-ground discharge leading to the first stroke to ground. The steps are color-coded with the first steps colored dark grey and the last steps colored light grey. Panel (a) represents the altitude of each newly added link. Shown are three views of the discharge tree: a perspective from (b) the x-z plane, (d) the x-y plane, (e) the y-z plane. (c) shows a histogram of the grid point occupied by a channel link taken as a function of altitude. The upper positive, central negative, and lower positive charge layers are shown as cylinders with diameter $2a_x$ and depth $2a_z$ outlined in grey. The initiation point of this discharge is beneath the upper positive layer and is marked by a cross. A triangle indicates the point at which the lightning tree reaches the ground.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

(Note that similar conclusions can be drawn for Figure 8: the portion of the positive cloud-to-ground discharge that develops in the positive charge layer is negative and the lower part which reaches the ground is positive.) However, a comparison of the behavior between the flashes from Figures 2 and 6 is quite tedious since Figure 2 also shows subsequent discharges to the ground. Nevertheless, it can be noted that in both cases, the propagation is essentially vertical, unlike the intracloud charges measured by Rison *et al.*[23] and modeled by Riousset *et al.*[10]. No LMA data are available for positive cloud-to-ground discharges, however, it can be noted that the altitude of initiation of the model at $\sim$9 km is in good agreement with measurements[32].

Rakov and Uman[4] reviews values of charge transfer due to the first stroke to the ground of both positive and negative cloud-to-ground discharges. In particular, negative cloud-to-ground discharges are measured to transfer 1.1 C to 24 C in the first stroke to ground. From our model we estimate a charge transfer of $\sim$11.6 C, which is consistent with previously cited values. Similarly, Rakov and Uman[4] cite values between 20 C to 80 C for the typical charge transfer corresponding to positive cloud-to-ground discharges. From the simulation results, we infer a charge transfer of $\sim$13.92 C, which is lower that the referred range, but still in reasonable agreement.

The linear charge density of the discharge channel can also be derived as $(|Q^+_{cha}| + |Q^-_{cha}|)/l$ (where Q^+_{cha} and Q^-_{cha} are the net charges carried by the positive and negative leaders, respectively), and compared to measurements. The linear charge density of the model negative cloud-to-ground channel is derived as $\sim$1.35 mC/m, which lies in the lower part of the scale (0.7 mC/m to 8.7 mC/m) quoted by Rakov and Uman[4]. The same derivation applied to the model positive cloud-to-ground channel yields a linear charge density of $\sim$3.2 mC/m. This value is comparable to the average linear charge density for leaders of $\sim$1 mC/m found by Helsdon *et al.*[18]. Although this value is not specific to positive cloud-to-ground discharges, it is a reasonable comparison considering the absence of available information on this issue.

Finally, we notice that Figure 7 shows a net field reduction of approximately 65% induced by the negative cloud-to-ground discharge, which is consistent with the previous modeling of intracloud discharges by Riousset *et al.*[10] and data reported by Winn and Byerley[33].

CONCLUSIONS

This paper makes several contributions to the fields of atmospheric electricity and gas discharge modeling, which can be summarized as follows:

- The ability of the lightning model described in Riousset *et al.*[10] to simulate both negative and positive cloud-to-ground discharges has been demonstrated by comparisons of simulation results to LMA data and other relevant data available in the referred literature.

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

- The initiation of cloud-to-ground discharges has been shown to be easier with configurations involving layers of non-uniform Gaussian charge density rather than with configurations employing disks of uniform charge density for the same amount of net charges in each layer.

ACKNOWLEDGMENTS

This material is based upon work supported by the National Science Foundation under Grants No. EEC-0244030 and ATM-0134838 to the Pennsylvania State University.

REFERENCES

[1] T. C. Marshall and M. Stolzenburg, “Electrical energy constraints on lightning,” *J. Geophys. Res.*, **107** (D7) 4052, doi:10.1029/2000JD000024 (2002).

[2] H. W. Kasemir, “A contribution to the electrostatic theory of a lightning discharge,” *J. Geophys. Res.*, **65** (7) 1,873–1,878 (1960).

[3] A. S. Jursa, ed., Handbook of Geophysics and the Space Environment, US Air Force Geophysics Lab., Springfield, VA, 1985.

[4] V. A. Rakov and M. A. Uman, Lightning Physics and Effects, Cambridge Univ. Press, New York, NY, 2003.

[5] L. Niemeyer, L. Ullrich and N. Wiegart, “The mechanism of leader breakdown in electronegative gases,” *IEEE Trans. on Electr. Insul.*, **24** (2) 309–324, doi: 10.1109/14.90289 (1989).

[6] B. B. Mandelbrot, The Fractal Geometry of Nature, updated and augmented ed., W. H. Freeman, New York, NY, 1983.

[7] N. I. Petrov and G. N. Petrova, “Physical mechanisms for intracloud lightning discharges,” *Techn. Phys.*, **38** (4) 287–290 (1993).

[8] E. R. Mansell, D. R. MacGorman, C. L. Ziegler and J. M. Straka, “Simulated three-dimensional branched lightning in a numerical thunderstorm model,” *J. Geophys. Res.*, **107** (D9) 4075, doi:10.1029/2000JD000244 (2002).

[9] D. P. Agoris, V. P. Charalambakos, E. Pyrgloti and S. Grzybowski, “A computational approach on the study of Franklin rod height impact on striking distance using a stochastic model,” *J. El. Stat.*, **60** (2–4) 175–181, doi: 10.1016/j.elstat.2004.01.020 (2004).

[10] J. A. Riousset, V. P. Pasko, P. R. Krehbiel, R. J. Thomas and W. Rison, “Three-dimensional fractal modeling of intracloud lightning discharge in a New Mexico

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

thunderstorm and comparison with lightning mapping observations," *J. Geophys. Res.*, doi:10.1088/xxx-xxxx, In review (2006).

[11] J. A. Riousset, Fractal modeling of lightning, Master's thesis, The Pennsylvania State University, University Park, PA (2006).

[12] L. M. Coleman, T. C. Marshall, M. Stolzenburg, T. Hamlin, P. R. Krehbiel, W. Rison and R. J. Thomas, "Effects of charge and electrostatic potential on lightning propagation," *J. Geophys. Res.*, **108** (D9) 4298, doi: 10.1029/2002JD002718 (2003).

[13] M. A. Uman, Lightning, reprint ed., Dover, Mineola, NY, 1984.

[14] V. P. Pasko, "Theoretical modeling of sprites and jets," in: M. Füllekrug, E. A. Mareev and M. J. Rycroft, ed., "Sprites, Elves and Intense Lightning Discharges," vol. 225 of *NATO Science Series II: Mathematics, Physics and Chemistry*, pp. 253–311, Springer, Heidleberg, Germany, 2006.

[15] E. M. Bazelyan and Y. P. Raizer, Lightning Physics and Lightning Protection, IoP, Philadelphia, PA, 2000.

[16] I. Gallimberti, G. Bacchiega, A. Bondiou-Clergerie and P. Lalande, "Fundamental processes in long air gap discharges," *C. R. Physique*, **3** (10) 1,335–1,359, doi:10.1016/S1631-0705(02)01414-7 (2002).

[17] V. Mazur and L. H. Ruhnke, "Model of electric charges in thunderstorms and associated lightning," *J. Geophys. Res.*, **103** (D18) 23,299–23,308 (1998).

[18] J. H. Helsdon, Jr., G. Wu and R. D. Farley, "An intracloud lightning parameterization scheme for a storm electrification model," *J. Geophys. Res.*, **97** (D5) 5,865–5,884 (1992).

[19] P. Krehbiel, W. Rison, R. Thomas, T. Marshall, M. Stolzenburg, W. Winn and S. Hunyady, "Thunderstorm charge studies using a simple cylindrical charge model, electric field measurements, and lightning mapping observations," *Eos Trans. AGU*, **85** (47), Fall Meet. Suppl., Abstract AE23A-0843 (2004).

[20] L. Niemeyer, L. Pietrono and H. J. Wiesmann, "Fractal dimension of dielectric breakdown," *Phys. Rev. Lett.*, **52** (12) 1,033–1,036, doi: 10.1103/PhysRevLett.52.1033 (1984).

[21] N. I. Petrov and G. N. Petrova, "Simulation of the branching and bending of breakdown channels in dielectrics," *Tech. Phys. Lett.*, **18** (2) 65–67 (1992).

[22] E. R. Williams, "The tripolar structure of thunderstorms," *J. Geophys. Res.*, **94** (D11) 13,151–13,167 (1989).

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)

[23] W. Rison, R. J. Thomas, P. R. Krehbiel, T. Hamlin and J. Harlin, “A GPS-based three-dimensional lightning mapping system: Initial observations in central New Mexico,” *Geophys. Res. Lett.*, **26** (23) 3,573–3,576, doi:10.1029/1999GL010856 (1999).

[24] T. C. Marshall, M. Stolzenburg, C. R. Maggio, L. M. Coleman, P. R. Krehbiel, T. Hamlin, R. J. Thomas and W. Rison, “Observed electric fields associated with lightning initiation,” *Geophys. Res. Lett.*, **32** L03813, doi: 10.1029/2004GL021802 (2005).

[25] D. R. MacGorman and W. D. Rust, The Electrical Nature of Storms, Oxford Univ. Press, New York, NY, 1998.

[26] V. A. Rakov, “A Review of Positive and Bipolar Lightning Discharges,” *Bulletin of the American Meterological Society*, **84** (96) 767–776, doi:10.1175/BAMS-84-6-767 (2003).

[27] N. Liu and V. P. Pasko, “Effects of photoionization on propagation and branching of positive and negative streamers in sprites,” *J. Geophys. Res.*, **109** A04301, doi:10.1029/2003JA010064 (2004).

[28] V. P. Pasko and J. J. George, “Three-dimensional modeling of blue jets and blue starters,” *J. Geophys. Res.*, **107** (A12) 1458, doi:10.1029/2002JA009473 (2002).

[29] N. Femia, L. Niemeyer and V. Tucci, “Fractal characteristics of electrical discharges: experiments and simulation,” *J. Phys D: Appl. Phys.*, **26** (4) 619–627, doi:10.1088/0022-3727/26/4/014 (1993).

[30] L. Niemeyer and H. J. Wiesmann, “Modeling of leader branching in electronegative gases,” pp. 134–139, Gaseous Dielectrics V: Proceedings of the Fifth International Symposium on Gaseous Dielectrics: Knoxville, TN, 3-7 May (1987).

[31] E. R. Williams, C. M. Cooke and K. A. Wright, “Electrical discharge propagation in and around space charge clouds,” *J. Geophys. Res.*, **90** (D4) 6,059–6,070 (1985).

[32] M. A. Uman, The Lightning Discharge, unabridged ed., Dover, Mineola, NY, 2001.

[33] W. P. Winn and I. Byerley, L. G., “Electric field growth in thunderclouds,” *Quart. J. R. Met. Soc.*, **101** 979–994 (1975).

Annual Research Journal, Vol. IV
Electrical Engineering Research Experience for Undergraduates

ISBN 0-913260-06-1 (http://www.ee.psu.edu/reu/)