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EQUIVALENCE PRINCIPLE APPLIED TO MODELING OF THUNDERSTORM CHARGE CONFIGURATION

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ABSTRACT

The equivalence principle has been used to model the changes in the electrical structure of a thunderstorm caused by lightning strikes.[1] However a rigorous justification of the applicability of this principle to model the electric field above the cloud remains to be established. This work uses an observation-based model of the electrical structure of thunderstorms to explore the effects of lightning on this structure and how related electric fields can be modeled using the equivalence principle. This study looks into applicability of the equivalence principle for the modeling of the electric field above the cloud. The electric field changes following a variation of charge before and after a lightning strike are calculated numerically and the results obtained with and without using the equivalence principle are compared. The results establish a proof of the validity of the equivalence principle for modeling the electric field change above the thundercloud following the occurrence of lightning.

INTRODUCTION

The first studies aimed to determine the electrical structure of thunderstorms started in the early 1900's.[2—6] The structure was initially believed to be a vertical dipole with a positively charged region above a negatively charged region. Around 1940[7,8] a tripole structure was adopted, with a lower positive charge region below the dipole. The two upper charge regions are considered to be the main charges and are specified to be approximately equal in magnitude. They form a positive dipole, i.e., the positive charge is above the negative, which gives

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an upward-directed dipole moment. The electric field intensity due to the three vertically stacked charges can be found by replacing the conducting ground with three image charges using the principle of superposition. The total electric field then becomes the vectorial sum of all six contributions. Because of the symmetry of the problem the magnitudes of the contributions from the actual charge and its images are equal at z = 0 km.[9] In 1996, it was suggested that the tripole and dipole charge structure models were too simplistic.[10,11] An improved model of the charge structure of the thunderstorm was in presented in 1998 by *Stolzenburg et al* as shown in Figure 1.[11] Yet, the tripole model appears to correctly represent the electrical structure in the convective region (see Figure 1).

Storms, similar to the one illustrate in Figure 1, commonly produce lightning, and so-called transient luminous events. Lightning is an electrical discharge between and within the three main charge regions of the cloud (lower positive, central negative, and upper positive charge regions) accompanied by charge transfer between the charge regions or to the ground. Transient luminous events (TLEs) are different types of electrical discharges produced by large thunderstorms in the altitude range 15–90 km, i.e, between the altitude of cloud tops and the lower ionosphere. The most common types of TLEs are blue jets, gigantic jets, sprites, and elves as illustrated in Figure 2 below.[12] Blue jets are relatively slow-moving fountains of blue light which emanate from the top of thunderclouds up to an altitude of 40 km at speeds ~100 km/s and are characterized by a blue conical shape.[12]

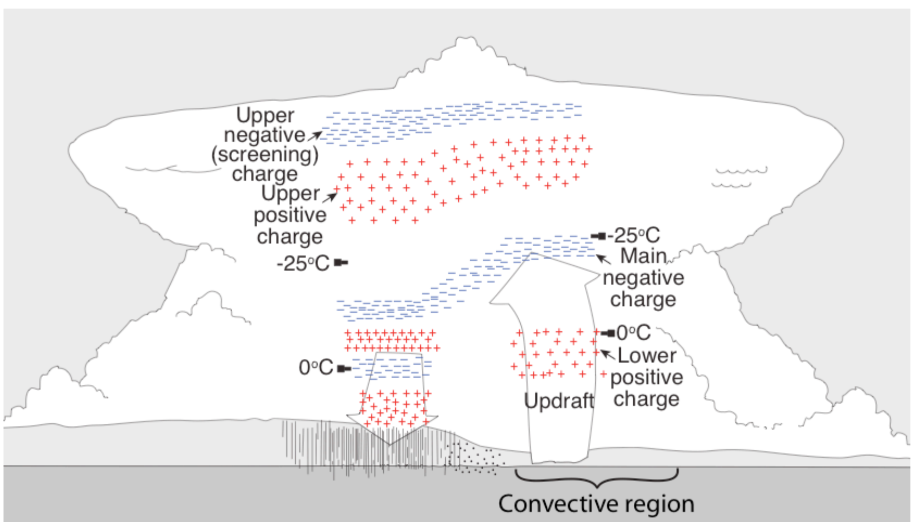

Figure 1: Schematic of charge structure of a thunderstorm.[11] A tripolar structure is visible in the convective region (right hand side) with a central negative, upper positive, and lower positive charge regions. The tripole is topped by an upper negative screening charge.

Gigantic jets are visually similar to blue jets and establish a direct path of electrical contact between thundercloud tops and the lower ionosphere.[12] Sprites are luminous glows occurring above thunderstorms at altitudes typically ranging from ~50 to 90 km. Sprites develop at the base of the ionosphere and move rapidly downward at speeds up to 10 000 km/s. In video, they exhibit a red color at their top, which gradually changes to blue at lower altitudes. The lateral extent of "unit" sprites is typically 5–10 km, and they last for several milliseconds.[1] Elves are rapid (<1 ms) optical emissions at 80–95 km altitudes with lateral extents up to 300 km.[1] Our study particularly applies to the investigation of the electric field that initiates the sprites.

The goal of this paper is to show how the equivalence principle can be used to represent the changes in the electrical structure of the thunderstorm induced by a lightning strike and the resulting variations of the electric field above the cloud up to the lower ionospheric region.[1] The principle of equivalence states that electric field change following the charge removal in a complex system of electrical charge regions can be modeled in the far field by the addition of an equivalent charge but with opposite polarity at the location of the original removal of charge. In this paper, we use the basic tripole charge structure of the cloud described above to model thunderstorm in order to demonstrate the applicability of this equivalence model in the framework of lightning and TLE studies.

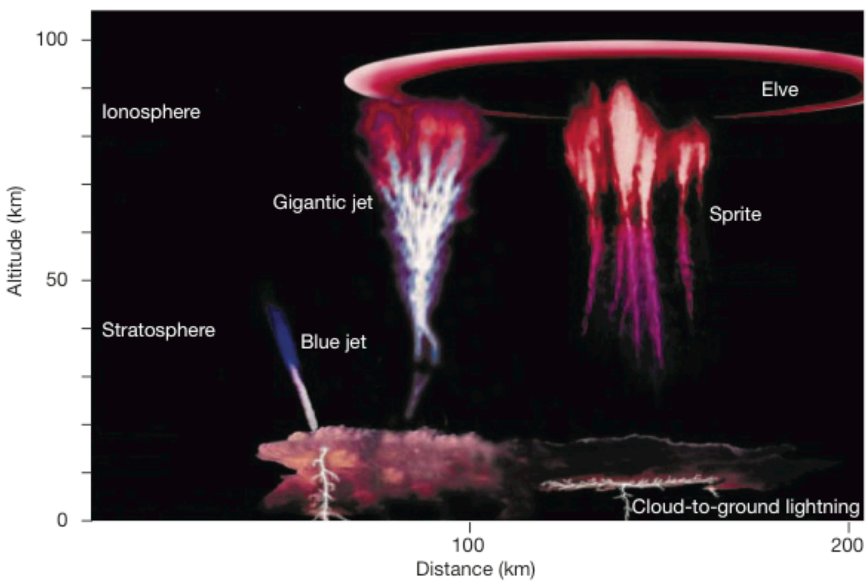

Figure 2: The different types of transient luminous events (TLEs) in relation to the altitude.[12]

MODEL FORMULATION

To test the equivalence principle in the framework thunderstorm studies, a classic tripolar charge structure[13] is used inside a dielectric cylinder of radius r_c and vertical extent z_c (Table I) that represent the limits of the cloud. Each charge layer in the structure is assumed to be cylindrically symmetric and disk shaped.[14] The dimensions, locations, and net charge contents of the charge regions for this test case are shown in Table II and were developed based on observations of a storm over Langmuir Laboratory, New Mexico on 31 July 1999.[14,15]

The value of the net charge content of each charge region is assumed prior a simulation run and reported in Table II. The source charges are uniformly distributed within each charge disk and produce time- and space-varying induced free charges ρ_f. The source charge densities ρ_s can be calculated by the assumed charge Q_i of the region i (positive, negative, lower positive charge layer) under consideration using the equation $\rho_{s,i} = \dfrac{Q_i}{\pi r_i^2 d_i}$ where the radius r_i and depth d_i of the region are given in Table I. Together ρ_s and ρ_f create an electric potential ϕ both inside and outside the storm. The basic set of equations relating ρ_f, ρ_s and ϕ are:

$$\nabla^2\phi = -\frac{\rho_s + \rho_f}{\varepsilon_o} \tag{1}$$

$$\frac{\partial \rho_f}{\partial t} - \nabla\sigma \cdot \nabla\phi = -\sigma\frac{\rho_s + \rho_f}{\varepsilon_o} \tag{2}$$

where σ is the atmospheric conductivity. The total charge density can be represented by $\rho_t = \rho_s + \rho_f$. The above equations express Gauss's law and conservation of charge, in which the conduction current $\boldsymbol{J}$ is assumed to be Ohmic and replaced by $\boldsymbol{J} = \sigma\boldsymbol{E} = -\sigma\nabla\phi$ in (2).[1,16]

The conductivity σ at any location (r, z) in the simulation domain is expressed by

$$\sigma(r,z) = \sigma_o e^{\frac{z+z_{gnd}}{h}}\left(1 - \frac{1-\tanh\left(\frac{r-r_c}{\alpha}\right)}{2} \times \frac{1-\tanh\left(\frac{z-z_c}{\alpha}\right)}{2}\right) \tag{3}$$

where the parameter α determines the thickness of the conductivity transition region between the cloud interior and the surrounding clear air (Table I). The conductivity inside the thunderstorm is reduced because of the larger value of the ion attachment coefficient in thunderclouds.[17]

Table I. Cloud Boundaries

	Symbol	Parameter [km]
Radius	r_c	5
Height	z_c	9
Boundary thickness	α	0.75

The conductivity outside the cloud increases exponentially with altitude z (term $\sigma_o e^{\frac{z+z_{gnd}}{h}}$ in (3)) with an altitude scaling factor h = 6 km and a conductivity at sea level σ_o defined as 5×10^{-14} S/m. [1,16] Inside the cloud the conductivity is reduced to zero by the factor in brackets following the exponential term $\left(1-\frac{1-\tanh\left(\frac{r-r_c}{\alpha}\right)}{2}\times\frac{1-\tanh\left(\frac{z-z_c}{\alpha}\right)}{2}\right)$ in (3) with a smooth transition at the boundary of width ~2α. [16] For the simulation presented in this paper α is set equal to 0.75 km.

In order to accurately reflect the effect of a negative cloud-to-ground discharge on the thunderstorm, the cloud-to-ground discharge is assumed to transfer –20 C to the ground from the central negative charge region as shown in Figure 3. The typical value of negative cloud-to-ground charge transfer ranges between 20–80 C.[9]

The time scale of the charge dissipation $\tau_\sigma(z)$ is given by $\tau_\sigma=\varepsilon_o/\sigma(z)$ where ε_o is the free space permittivity, $\sigma(z)$ is the conductivity of the medium as a function of the altitude z, and σ_o is given by (3). It is used to find the amount of time that the system needs to reach steady state, and consequently the time the model needs to run. At the top of the cloud τ_σ(12 km)~14.5 s and therefore the steady state is achieved after ~3τ_σ ≈43.5 s.

Table II. Charge Values, and Dimensions for Charge Regions Used in the Model

Charge Layer	Altitude [km]*	Depth [km]	Radius [km]	Charge [C]
Positive	6.75	1.5	4.0	45
Negative	3.75	1.5	3.0	-50
Lower Positive	2.00	1.5	1.5	-5

*above ground level (ground level set at 3 km)

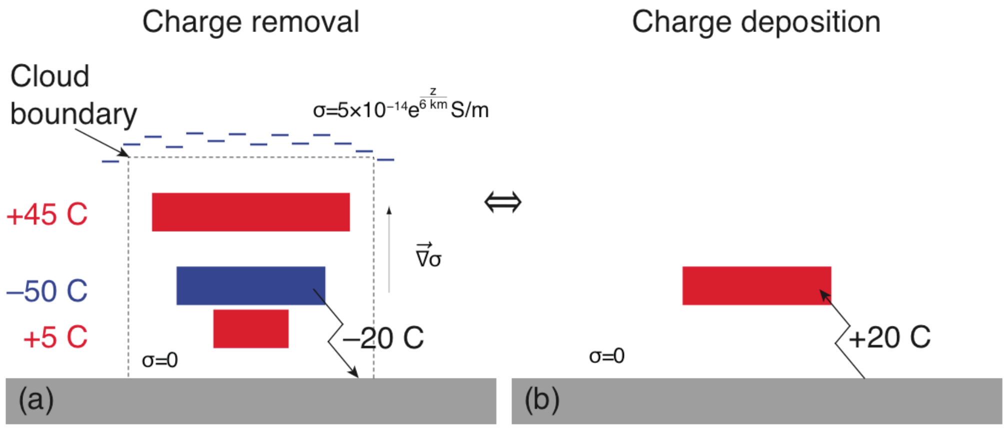

Figure 3: Schematic of the full and equivalent models. Panel (a) shows the thundercloud structure and the –20 C discharge from the central negative region. Panel (b) shows the equivalent model of the thunderstorm in which +20 C charge is deposited at the location from which –20 C charge was removed.

For this model $\tau_{\min} = \tau_{\sigma_{\max}}$ = 11.1 μs. To find a suitable time step to satisfy the Courant-Friedrick-Lewy Condition[16] for stability of numerical scheme δt was defined as $\delta t < \frac{1}{2}\tau_{\min} = \frac{1}{2}(11.1\mu s)$. For the run presented in this simulation, we choose δt = 0.5 μs.

The principle of equivalence, as defined for the purposes of studies presented in this paper, states that the removal of a –20 C charge from the midlevel negative charge region can be modeled by an addition of an equivalent +20 C charge deposited at the same location. This can be represented by a single positive charge region with dimensions of the central negative region from and at the same location (see Table II and the Figure 3a), with net charge content equal to +20 C, in a dielectric atmosphere as shown in Figure 3b. Unlike for the case of the modeling of a tripolar cloud, the absence of a conductive atmosphere leads to a static solution in that latter case.

RESULTS AND DISCUSSION

In this section we present and discuss the results from the simulation of the variations in electric field following a negative cloud-to-ground discharge removing –20 C from the negative charge region of a classic, normally electrified, tripolar thundercloud. Hereafter, we refer to this case as “full model case.” We also present the results from an alternative model based on the equivalence principle, and we refer to this second case as “equivalent model case.”

From the simulation results we can compare the total charge density of the full model case over the entire simulation domain right before and at the moment when the cloud-to-ground discharge occurs. Figure 4 shows the cloud right before the cloud-to-ground discharge at t = 37.5 s. Figure 4a shows the total charge density ρ_t just before the initiation of the cloud-to-ground discharge, while Figure 4b shows the magnitude of the electric field at the same instant of time. The Figure 4c shows the profiles of the source charge density ρ_s, induced free charge density ρ_f, and total charge density ρ_t at r = 0 km. Figure 5 shows the same results as Figure 4, except at the time when the discharge occurs. From these figures it is clear that the magnitude of the electric field in the vicinity of thundercloud decreases as the discharge occurs.

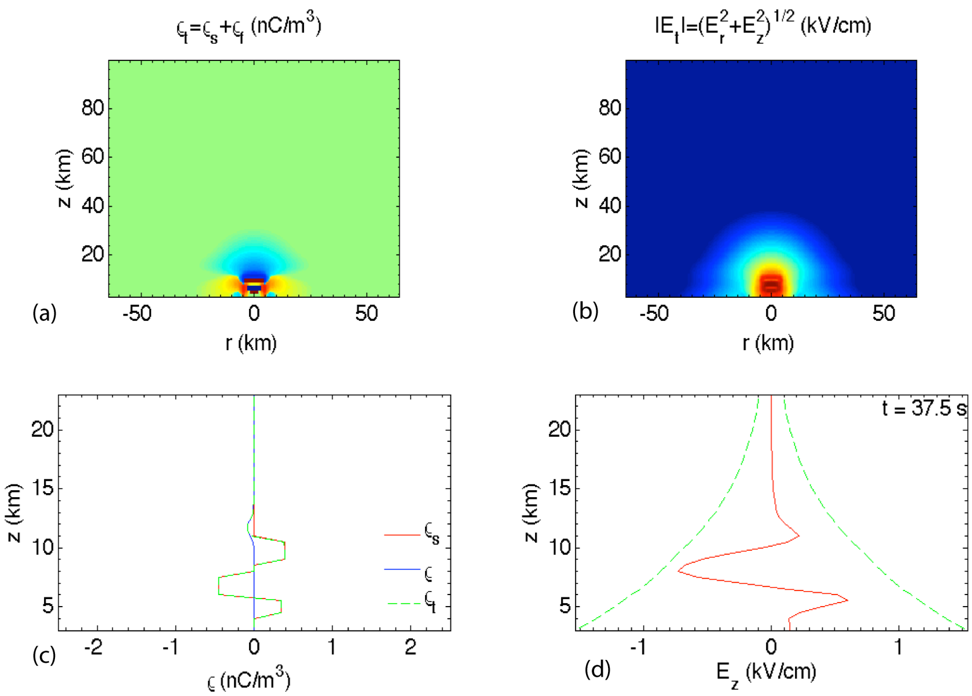

Figure 4: Simulation of full model case at t ~ 37.5 s. (a) Total charge density and (b) electric field magnitude, (c) source charge density ρ_s, induced free charge density ρ_f, and the total charge density $\rho_{t,}$ and (d) electric field with respect to altitude z, at r = 0 km.

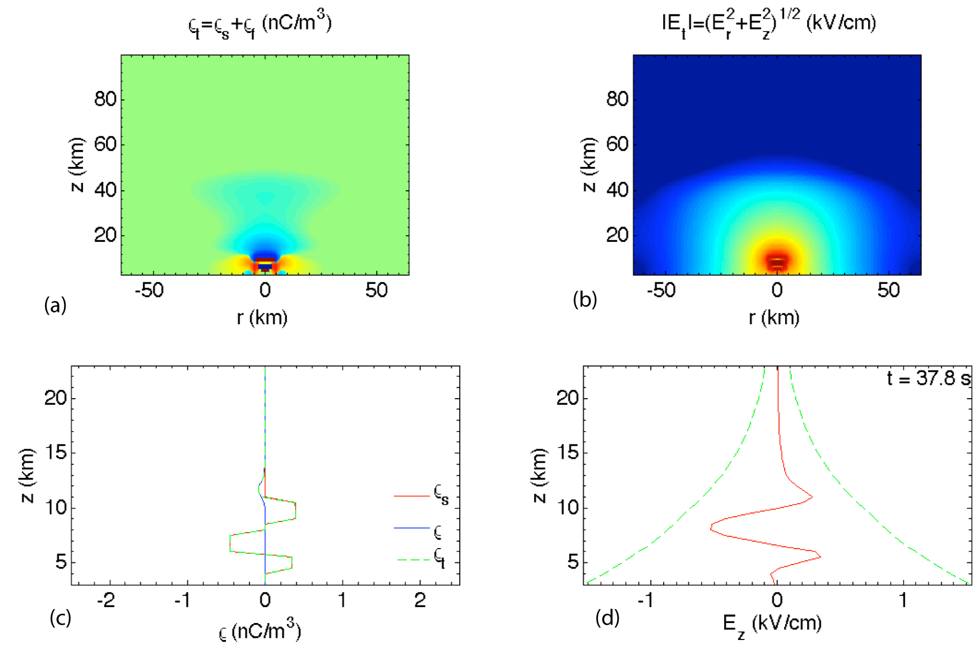

Figure 5. Same as Figure 4, except at t = 37.8s, as the –20 C cloud-to-ground discharge occurs.

Figure 6 shows the electric field with respect to the altitude of the full model case and the equivalent model on the same graph on the axis of the simulation domain at r = 0 km. From this figure it can be seen that at higher altitudes (i.e., above ~25 km) the electric field of the full model case and the equivalent model case are in excellent agreement. The figure also shows the lightning initiation threshold, which is the electric field, required to initiate the lightning.[14] At lower altitudes, the electric fields of the two models become significantly different. Figure 6 displays three local minima corresponding to the field inversions at the locations of the lower positive charge, central negative, and upper positive regions of the altitudes 5 km, ~7 km, and ~10 km, respectively. In contrast the equivalent model only presents one field inversion at the location of the unique positive charge (z~7km).

The equivalence principle is sometimes misinterpreted when applied to the charge in thunderclouds by implying that the excess of positive charge remaining in the cloud after the discharge is the source of the electrostatic field in the lower ionosphere and associated with the occurrence of some TLEs (in particular sprites).[18]

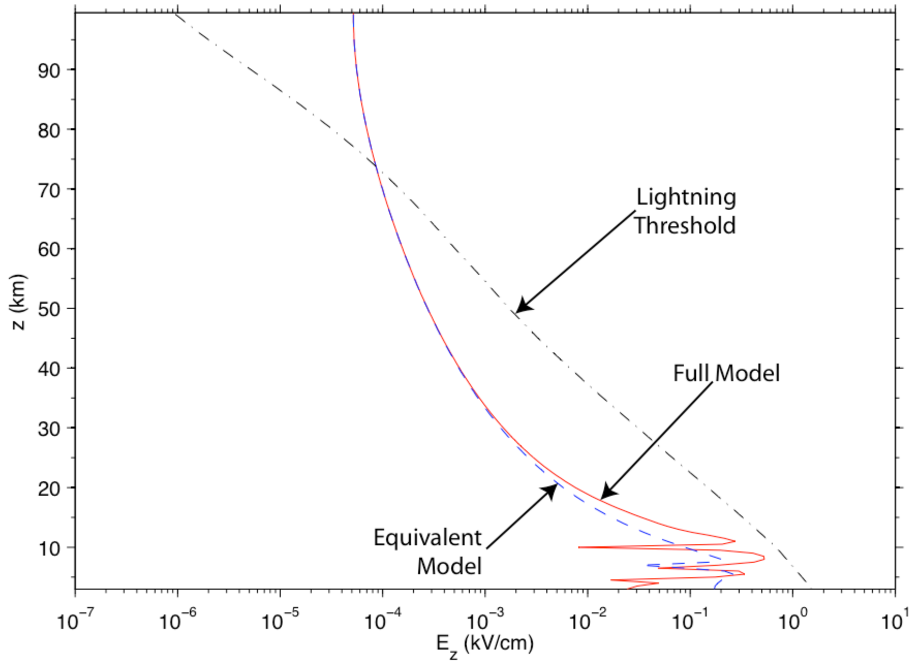

Figure 6: Electric field amplitude E_z as a function of the altitude z at $r = 0$ km. The solid line represents the full model case, the dotted line is the equivalent model and the dotted-dashed line shows the lightning initiation threshold assuming that it scales proportionally to air density with altitude.

The equivalence principle actually states that the removal of charge subsequent to the occurrence of a cloud-to-ground discharge is equivalent to the addition of a new charge of opposite polarity. *Pasko et al.*[1] applied the equivalence principle to simple, dipolar, normally electrified thundercloud. More specifically *Pasko et al.*[1] further showed that as the thundercloud charges slowly build up before a lightning discharge, high-altitude regions are shielded from the quasi-electrostatic fields of the thundercloud charges by space charge induced in the conducting atmosphere at lower altitudes. The appearance of this shielding charge is a consequence of the finite vertical conductivity gradient of the atmosphere above the thundercloud. When one of the thundercloud charges is quickly removed by a lightning discharge, the remaining charges of opposite sign in and above the thundercloud produce a large quasi-electrostatic field that appears at all altitudes above the thundercloud, and endures for a time equal to approximately the local relations time ($\tau_\sigma=\varepsilon_o/\sigma(z)$) at each altitude. This large electric field can be thought of as being the difference between the electrostatic field dictated (via Coulomb's law) by the dipole configuration of thundercloud charges and the polarization charge which are in effect before the discharge, and that required by the combination of the single thundercloud charge remaining after the discharge and polarization charge.[1] In cases of more complex charge distributions in the thundercloud which sometimes involve up to six charge layers in the vertical direction,[19] each of the charge centers can be viewed as generating its own polarization charge in and above thundercloud, and the resultant configuration of the electric field and charge density can be obtained by using principle of superposition. This consideration is helpful in visualization of the fact that the electric field appearing at mesospheric altitudes after the charge removal by cloud-to-ground lightning discharge is defined mostly by the absolute value and altitude of the removed charge and is essentially independent of the complexity of the charge configuration in the cloud. The charge removal can also be viewed as the "placement" of an identical charge of opposite sign. The initial field above the cloud is simply the free space field due to the "newly placed" charge and its image in the ground which is assumed to be perfectly conducting.[1] These conclusions are further supported and demonstrated by the simulation results presented in the present paper.

CONCLUSIONS

In this paper, we demonstrate how the equivalence principle can be used to represent the changes in the electrical profile of the thunderstorm induced by a lightning strike and the resulting variations of the electric field above the cloud up to the lower ionospheric region.[1] The model simulations for the full model case and the equivalent model case are shown to have very similar electric fields, especially when comparing the field farther away from the cloud. The full model case simulates a thundercloud that has a three charge region electric structure (lower positive, central negative, and upper positive charge layers) enclosed in the conductive atmosphere. The cloud has parameters of a 5 km radius, 9 km height, 0.75 km thickness of the cloud boundary, and is placed above the perfectly electrically conducting ground located at altitude of 3 km to simulate the condition of a New Mexico thunderstorm. The full model simulation showed a cloud-to-ground discharge removing –20 C from the negative charge region. The equivalent model simulated the equivalent system of the full model case in a non-conductive atmosphere. The agreement above 25 km between the full model, including a tripolar cloud in the conducting atmosphere, and the equivalent model as shown in Figure 6 demonstrates the applicability of the equivalence principle to model the electric field induced by a cloud-to-ground discharge in a realistic thundercloud model above altitudes 20–30 km.

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